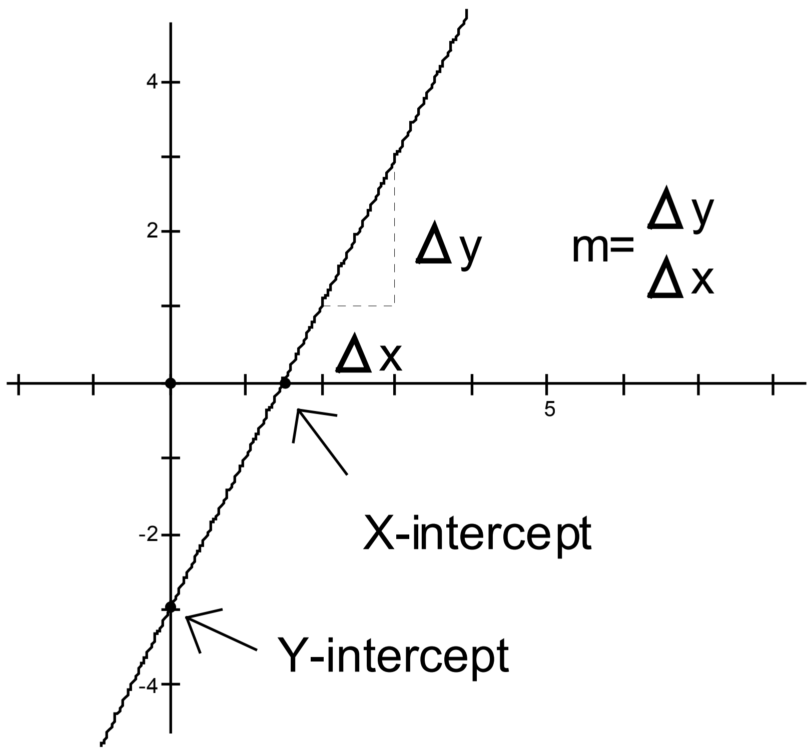
Section 1-10, Mathematics 108

Linear Equations

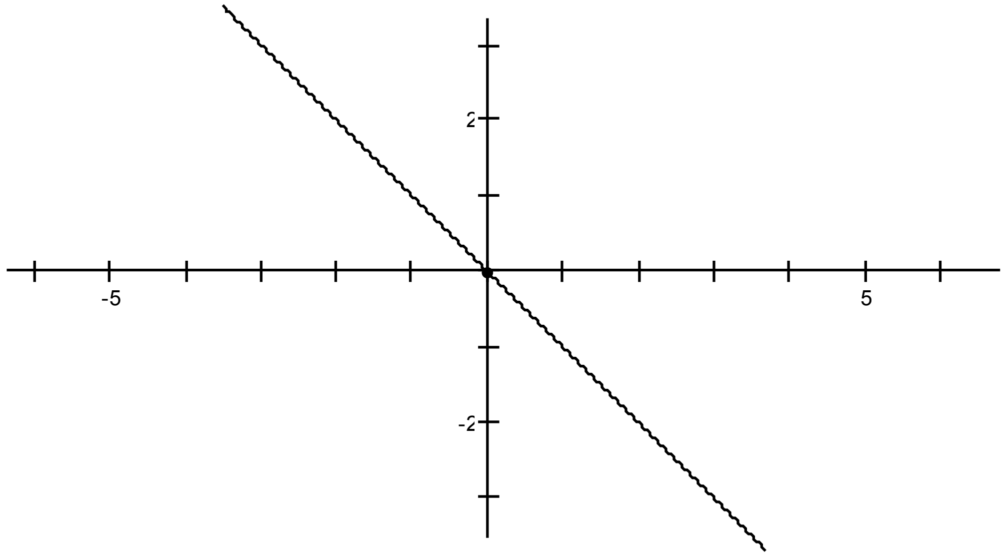
Slope Intercept Form

$y = mx + b$

In this form of a linear equation in x and y the constant $m = \frac{\Delta y}{\Delta x}$ is the slope of the line, and b is the Y intercept, the place where the line crosses the Y-axis.



Note: this line goes up from left to right so it has $m>0$.



This line goes down from left to right so $m<0$.

If $m=0$, the line has zero slope and is horizontal.

A vertical line has an undefined slope.
The equation of a vertical line will be of the form: $x = c$ where c is a constant.

Finding the intersection of two linear equations

Given two linear equations, there are a few different ways to find their point of intersection.

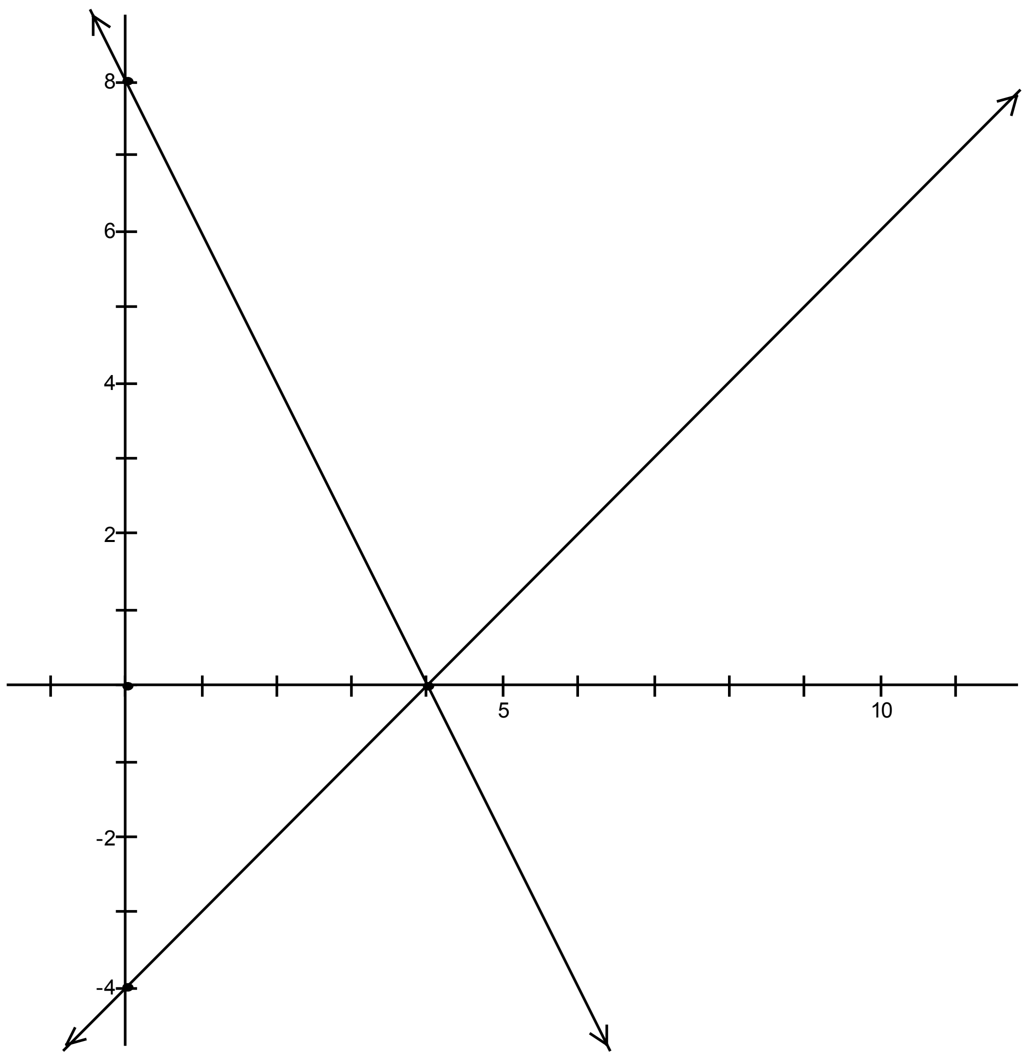
It's always a good idea to first check whether the two equations have same slope.
If the equations have the same slope then either they are parallel, or they are the same line.

Here's an set of equations that we will solve using four different methods:

$x - y = 4$

$2x + y = 8$

Solve by graphing, potentially imprecise.



In this case we can see that the intersection is (4,0) which we can check by plugging back into the two equations.

Solving by substitution

To solve by substitution, we get one equation in the form x= or y= and then substitute into the second equation:

$$x = y + 4$$
$$2(y+4) + y = 8$$
$$2y + 8 + y = 8$$
$$3y = 0$$
$$y = 0$$

Plugging y=0 into the first equation we get x=4 so we get the same answer (4,0).

Solving by elimination

To solve by elimination, you add or subtract the two equations in a way that eliminates one of the variables. You might have to first multiply one or both equations to accomplish this.

$$x - y = 4$$
$$2x + y = 8$$

Note that if we just add these two equations, the y will be eliminated:

$$x - y + 2x + y = 4 + 8$$
$$3x = 12$$
$$x = 4$$

If we plug x=4 into either equation and solve for y we find that y=0.

Cramer's Rule

Cramer's rule is like quadratic formula in that the solution is pre-computed abstractly. To use Cramer's rule you have to know that the determinant of a matrix is as follows:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Cramer's rule tells us that the solution to:

$$ax + by = c$$
$$dx + ey = f$$

$$x = \frac{\begin{vmatrix} c & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

Note that both denominators are the same. If the denominator is zero, the lines are parallel or the same. Let's check our problem one last time:

$$x - y = 4$$
$$2x + y = 8$$

$$x = \frac{\begin{vmatrix} 4 & -1 \\ 8 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}} = \frac{4+8}{1 - {}^{-}2} = \frac{12}{3} = 4$$

$$y = \frac{\begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}} = \frac{8-8}{1 - {}^{-}2} = 0$$

Given two points, find the equation of the line that passes through them

$(2,1),(8,5)$

First find the slope

$$m=\frac{\triangle y}{\triangle x}=\frac{5-1}{8-2}=\frac{4}{6}=\frac{2}{3}$$

The equation of the line is then $y=\frac{2}{3}x+b$

To find *b* substitute either of the two points and solve for *b*

$$1=\frac{2}{3}(2)+b$$
$$1=\frac{4}{3}+b$$
$$b=1-\frac{4}{3}=-\frac{1}{3}$$

So the equation is

$$y=\frac{2}{3}x-\frac{1}{3}$$

An alternative is to use the **Point-Slope form of a line**

$$y-y_1=m(x-x_1)$$
or
$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

Vertical and Horizontal lines

Note the methods mentioned above may break down in the case of horizontal or vertical lines.

The equation of a horizontal line is $y=c$ where c is a constant and

the equation of a vertical line is $x=d$ where d is a constant.

Parallel and Perpendicular lines

Two lines with undefined slope will be parallel.

If one line has zero slope and the other has undefined slope, the two lines will be perpendicular.

Otherwise lines with the same slope will be parallel.

Perpendicular lines have **negative-inverse** slopes.

Example:

Find the equation of a line parallel to $4x+6y+5=0$ that goes through the point $(5,2)$.

Putting the equation into slope intercept form:

$y=-\frac{2}{3}x-\frac{5}{6}$ we find that

$m=-\frac{2}{3}$

So the equation of the new line is

$$y=-\frac{2}{3}x+b$$

Plugging in (5,2) we solve for *b*

$$2=-\frac{2}{3}(5)+b$$

$$b=2+\frac{10}{3}=\frac{16}{3}$$

So the new equation is

$$y=-\frac{2}{3}x+\frac{16}{3}$$

Example:

Find the equation of the line perpendicular to the line

$4x+6y+5=0$ that goes through the point $(0,0)$.

The slope of the first line is $m=-\frac{2}{3}$ so the new line which is perpendicular has a negative reciprocal slope:

$$m=-\frac{1}{\left(-\frac{2}{3}\right)}=\frac{3}{2}$$

The equation of the new line is then

$$y=\frac{3}{2}x+b$$

Plugging in (0,0) and solving for *b* we find

$$0=\frac{3}{2}(0)+b$$
$$b=0$$

So the solution is

$$y=\frac{3}{2}x$$